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On steady non-breaking downstream waves and the wave resistance - Stokes' method

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Abstract

© 2017 Cambridge University Press. In this work, we have obtained explicit analytical formulae expressing the wave resistance of a two-dimensional body in terms of geometric parameters of nonlinear downstream waves. The formulae have been constructed in the form of high-order asymptotic expansions in powers of the wave amplitude with coefficients depending on the mean depth. To obtain these expansions, the second Stokes method has been used. The analysis represents the next step of the research carried out in Maklakov & Petrov (J. Fluid Mech., vol. 776, 2015, pp. 290-315), where the properties of the waves have been computed by a numerical method of integral equations. In the present work, we have derived a quadratic system of equations with respect to the coefficients of the second Stokes method and developed an effective computer algorithm for solving the system. Comparison with previous numerical results obtained by the method of integral equations has been made.

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Keywords

channel flow, surface gravity waves, waves/free-surface flows

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